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Modeling Time Series with Asymmetric Volatility and Long Memory

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ABSTRACT

Time series modeling of the price of agricultural commodities has immense importance in the Indian agricultural landscape. Volatility is an intrinsic property of time series. If positive and negative shocks of the same scale have differing effects on it, it is said to be asymmetric. The volatility of any time series is said to have long-term persistence if, for any given time epoch, it is significantly influenced by its distant past counterpart. The fractionally Integrated Exponential Generalized Autoregressive Conditional Heteroscedastic (FIEGARCH) model may be used to capture asymmetric volatility in any time series with long-term persistence. This paper uses the modal price series of onion for Delhi, Lasalgaon, and Bengaluru markets and S&P 500 index (close) data for empirical illustration. The GARCH, EGARCH, FIGARCH, and FIEGARCH models have been applied to the selected data sets. Significant asymmetric and long term persistence volatility in the selected time series has been found. It has been observed that the FIEGARCH model outperformed the other models in capturing volatility for all the selected time series.

Keywords: GARCH; Long memory; Nonlinear models; Time series; Volatility

JEL Codes: C22, C32, C51, C53, Q02, Q13

I

INTRODUCTION

A time series refers to a sequence of data points recorded at successive points at regular intervals over a specific period. Time series analysis allows us to make informed decisions based on historical patterns. A unique feature of a time series is that the successive realizations separated through time are correlated. This important feature helps to capture the underlying phenomenon of a time series. A time series can be decomposed into linear and nonlinear components in a broad perspective. Time series modeling has gained importance since the 1970s and was pioneered by Box and Jenkins (Box *et al*., 2015) through the introduction of the autoregressive integrated moving average (ARIMA) model under the assumption of linearity and stationarity of the data set and the homoscedasticity of the error variance. Many applications of the ARIMA methodology can be found in the literature (Paul & Das, 2013; Paul *et al*., 2020). The ARIMA methodology bears the lacuna of only addressing the linear component of a time series. To capture the nonlinear component of a time series like volatility, various complex models are introduced over time. Volatility is an important aspect of time series modeling, which represents the phenomenon of unexpected variation in the realizations of a time series. Proper knowledge of the behaviour of volatility of a financial time series can be helpful to all the stakeholders dealing with

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it. Engle (1982) proposed the autoregressive conditional heteroscedastic (ARCH) model by relaxing the assumption of linearity and homoscedasticity of error variance to capture the volatility. Later, Bollerslev (1986) and Taylor (1986) proposed the generalized ARCH (GARCH) model independently of each other. Many applications of the ARCH and GARCH models can be found in the literature (Paul *et al*., 2009, 2015 a). The GARCH architecture exhibits the same effect on volatility due to the positive and negative shocks of the same magnitude. This type of symmetric characterization may not be feasible for many practical situations. The reverse situation is regarded as asymmetric volatility when positive and negative shocks of the same magnitude affect volatility differently. The exponential GARCH (EGARCH) model (Nelson, 1991) is a better alternative to the GARCH model for addressing asymmetric volatility (Ghosh *et al*., 2010; Rakshit *et al*., 2023). Again, there may be long-term persistency among the realizations of any time series. The autocorrelation function (ACF) and partial autocorrelation function (PACF) are significant for a long lag (hyperbolic decay) in a long memory process. Long-term persistency can be found in linear dynamics (mean model) and nonlinear dynamics (variance model). To model the long-term persistency in the mean structure, Granger and Joyeux (1980) introduced the autoregressive fractionally integrated moving average (ARFIMA) model, where the differencing parameter is associated with fractional values instead of integer values like the ARIMA model. Similarly, the fractionally integrated term is introduced in the GARCH model to obtain the fractionally integrated GARCH (FIGARCH) model (Baillie *et al*., 1996) for capturing the long-term persistency in volatility. Application of the ARFIMA model (Paul, 2014), FIGARCH model (Paul *et al*., 2016 a; Rakshit & Paul, 2023), and the hybrid ARFIMA-FIGARCH model (Mitra *et al*., 2018) in agricultural price series can be found in the literature. Bollerslev and Mikkelsen (1996) proposed the fractionally integrated EGARCH (FIEGARCH) model to capture asymmetric volatility in long-term persistence.

Prices for agricultural commodities also exhibit volatility. The reasons for volatility may be due to the seasonal production cycle (Karali & Thurman, 2010), weather abnormalities, a sudden disruption in the supply chain (Paul & Yeasin, 2022; Kumar *et al*., 2023), and policy changes. Onion is India's second most produced vegetable after potatoes (31.273 million tonnes as per 3rd Advance Estimates, 2021-22). Onion price exhibits a high degree of price volatility. The entire onion supply chain is heavily dependent on government regulations and is also disrupted frequently due to weather abnormalities. The paucity of proper storage infrastructure is another cause of price volatility. Various publications highlighted the different aspects of onion price volatility in India (Paul *et al*., 2015 b; Paul *et al*., 2016 b; Saxena *et al*., 2020; Rakshit *et al*., 2021). There exists a strong element of uncertainty in the market arrivals of onions, which, as a result, causes onion price volatility in India. This paper uses the modal price series of onion for Delhi, Lasalgaon, and Bengaluru markets and S&P 500 index (close) data. The GARCH, EGARCH, FIGARCH, and FIEGARCH models have been applied to the selected time series. The rest of the paper has been organised as follows: Section II includes the theoretical aspects of this study; Section III includes the empirical illustration, followed by concluding remarks in the final section.

II

MATERIALS AND METHODS

2.1 The ARCH and GARCH Models

Linear models, such as ARIMA, cannot describe changes in conditional variance structure in data due to their assumption of homoscedasticity in the error variance. The ARCH model works by accounting for significant autocorrelations in the squared residual series.

A process $\{\varepsilon_t\}$ is said to follow the ARCH (q) model if the conditional distribution of $\{\varepsilon_t\}$ given the available information (ψ_{t-1}) up to $t-1$ time epoch is represented as:

$$\varepsilon_t|\psi_{t-1} \sim N(0, h_t) \text{ and } \varepsilon_t = \sqrt{h_t}\nu_t \qquad \text{…. (1)}$$

Where ν_t is identically and independently distributed (IID) innovation with zero mean and unit variance. According to time series data, the distribution of innovation varies. Generalized Error Distribution (GED) is an alternative when the data does not follow normal. The conditional variance h_t of ARCH (q) model is calculated as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \alpha_0 > 0, \alpha_i \geq 0 \;\forall\; i \text{ and } \sum_{i=1}^{q} \alpha_i < 1 \qquad \text{… (2)}$$

To achieve a reasonable level of model precision, the ARCH model necessitates estimating many parameters. The GARCH model is a more parsimonious version of the ARCH model, where the number of parameters to be estimated is smaller. The conditional variance is treated as a linear function of its lags in the GARCH model. The GARCH (p, q) model has the following form of conditional variance

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

provided $\alpha_0 > 0, \alpha_i \geq 0 \;\forall\; i, \beta_j \geq 0 \;\forall\; j$ ….(3)

α_i and β_j parameters indicate how previous shocks and volatility have influenced current volatility, respectively. The GARCH (p, q) model is said to be weakly stationary if and only if

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \qquad \text{….. (4)}$$

The GARCH model assumes that the magnitude of the shocks determines the effect of volatility and that the sign of the shocks has no effect. The EGARCH model can help overcome this lacuna.

2.2 *EGARCH Model*

The EGARCH model has been defined by describing the conditional variance in terms of the logarithm function. Aside from undertaking asymmetric volatility, the primary benefit of the EGARCH model over the GARCH model is that no restrictions have been levied on the model's parameters because the positivity of the conditional variance is always achieved. The conditional variance for the EGARCH model has been defined as

$$\ln h_t = \alpha_0 + \sum_{j=1}^{p} \beta_j \ln h_{t-j} + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right) \qquad \ldots. (5)$$

where, γ_i is the asymmetric factor that explains the asymmetric effect due to external shocks.

2.3 The FIGARCH Model

The FIGARCH model is useful when the volatility is symmetric and the volatility exhibits long-term persistence. After some algebraic operations, the FIGARCH model has been derived by introducing a fractional differencing parameter in the GARCH model. Tayefi & Ramanathan (2012) thoroughly reviewed the FIGARCH model. The conditional variance equation of GARCH (p,q) has been given by

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad \ldots. (6)$$

This representation can also be expressed as an equivalent ARMA-type representation as

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \varepsilon_{t-j}^2 - \sum_{j=1}^{p} \beta_j z_{t-j} + z_t \qquad \ldots. (7)$$

where $z_t = \varepsilon_t^2 - h_t = h_t v_t^2 - h_t = (v_t^2 - 1) h_t$

This equation can be expressed as an ARMA (m, p) process in ε_t^2 as

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \alpha_0 + [1 - \beta(L)] z_t \qquad \ldots. (8)$$

where, $m = max\{p, q\}$. This $\{z_t\}$ process can be regarded as an innovation for conditional variance. From this ARMA (m, p) process equation, the integrated GARCH (p,q) process can be defined as

$$[1 - \alpha(L) - \beta(L)](1 - L)\varepsilon_t^2 = \alpha_0 + [1 - \beta(L)] z_t \qquad \ldots. (9)$$

The FIGARCH model can be obtained by replacing the first difference operator $(1 - L)$ with the fractional differencing operator $(1 - L)^d$, where d is a fraction $0 < d < 1$. Here, the long memory operator has been applied to the squared errors. Hence, the FIGARCH(p, d, q) model can be expressed as

$$[1 - \alpha(L) - \beta(L)](1 - L)^d \varepsilon_t^2 = \alpha_0 + [1 - \beta(L)] z_t \qquad \ldots. (10)$$

2.4 The FIEGARCH Model

The FIEGARCH model is effective when volatility is asymmetric and has long-term persistence. This model has been developed by performing certain algebraic operations on the conditional variance equation of the EGARCH model and incorporating the fractional differentiation component. The FIEGARCH model, like the EGARCH model, has no parameter restrictions. The conditional variance h_t of the FIEGARCH (p, d, q) model has been defined as follows,

$$\beta(L)(1-L)^d(\ln h_t - \alpha_0) = \alpha(L)g(z_{t-1})$$
$$\Rightarrow \ln h_t = \alpha_0 + \frac{\alpha(L)}{\beta(L)}(1-L)^{-d}g(z_{t-1}) \quad \text{.... (11)}$$

where, $g(z_{t-1}) = \theta z_{t-1} + \gamma(|z_{t-1}| - E|z_{t-1}|)$ and $z_{t-1} = \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$ is the normalized innovation. $\beta(L)$ and $\alpha(L)$ are polynomials in the lag operator, such that $\beta(L) = (1-\beta_1 L) \times ... \times (1-\beta_p L)$ and $\alpha(L) = (1+\alpha_1 L) \times ... \times (1+\alpha_q L)$.

III

EMPIRICAL ILLUSTRATION

3.1 Data Description

For empirical illustration purposes, the daily modal spot prices (Rs./q) of onion for three important markets, namely Delhi, Lasalgaon, and Bengaluru, have been collected from the Ministry of Agriculture & Farmers' Welfare, Government of India for the study period of 1st January 2008 to31st December 2023. Besides the onion price data, the S&P 500 index (close) data for the above-mentioned period has also been collected from the website of Yahoo Finance. The daily series has been chosen because it includes many realizations over a long period, which increases the likelihood of long-term persistence. Since the square of return has been regarded as the realization of volatility, the analysis has been continued with the log return series of the selected time series data. For a time series $\{y_t\}$ the log return series $\{r_t\}$ has been calculated as

$$r_t = \ln \frac{y_t}{y_{t-1}} \quad \text{.... (12)}$$

The latest 250 realizations of the log return series of each selected time series have been utilised as the model validation set, while the entire preceding section of the series has been used as the model building set.

3.2 Descriptive Statistics

Table 1 shows the descriptive statistics of the selected onion price series and the S&P 500 index data. There are 5844 realizations for the price series and 4027 realizations for the S&P 500 index data. The number of observations for the selected price series and the S&P 500 index are different. No data is available for the S&P 500 index for Saturday and Sunday as the financial market remains closed on the weekend. For the onion price series, the missing observations were imputed by the last observation carried forward (LOCF) method. In onion price, the Bengaluru market has the highest mean price, while the Lasalgaon market has the lowest. The same pattern has been seen for the minimum price. The maximum price in the Bengaluru market is significantly higher than in the Lasalgaon and Delhi markets. The Delhi market has the highest median price, followed by Bengaluru and Lasalgaon markets. All the chosen time series have a considerably high degree of variance. Lasalgaon and Bengaluru markets have nearly the same coefficients of variation (C.V.). The S&P 500 index has a much lower C.V. than the selected onion price series. The skewness and kurtosis of the S&P 500 index are also much lower than the other three price series. The kurtosis of the Bengaluru market price series is significantly higher than the others.

TABLE 1: DESCRIPTIVE STATISTICS OF DAILY PRICE SERIES OF THE SELECTED ONION MARKETS AND S&P 500 INDEX

Statistics (1)	Delhi (2)	Lasalgaon (3)	Bengaluru (4)	S&P 500 (5)
Mean (Rs./q)	1371.79	1350.29	1400.43	2353.16
Median (Rs./q)	1105.00	1030.50	1100.00	2087.79
Minimum (Rs./q)	275.00	230.50	300.00	676.53
Maximum (Rs./q)	7650.00	8625.00	12500.00	4796.56
S.D. (Rs./q)	892.20	1031.55	1059.42	1112.93
CV (%)	65.04	76.39	75.65	47.30
Skewness	2.05	2.01	3.15	0.59
Kurtosis	5.40	5.33	17.91	-0.83

Figure 1 shows the time plots of the selected time series. The time plots of all the price series show a similar pattern of price change. This indicates that almost the same set of causes has ruled the price movement throughout the country. Massive price increases occurred in 2010, 2013, 2017, 2019, 2020, and 2023. The greatest ever price increase in history occurred towards the end of 2019. The bottom most time plot is for the S&P 500 index. The time plot of the S&P 500 index demonstrates a general upward tendency throughout time.

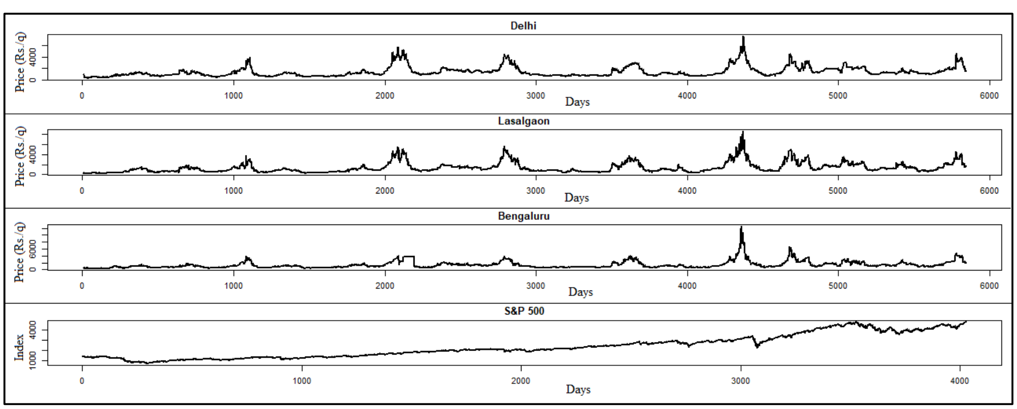

Figure 1: Time Plots of the Daily Price Series of Selected Onion Markets and the S&P 500 Index

3.3 Test for Normality

The normality of all the selected time series and the corresponding log return series have been tested using the Jarque-Bera test (Jarque & Bera, 1980). For this test, the null hypothesis is H_0: The series follows normal distribution; against the alternative hypothesis H_1: The series does not follow normal distribution. It has been seen that (Table 2) all the selected series and their corresponding log return series do not follow the normal distribution at a 1% level of significance. The kernel density plots (Figure 2) also support the same. The log return series of the selected time series has been thought to follow the GED. Hence, the distribution of innovation has been considered as the GED.

TABLE 2: JARQUE-BERA TEST

Series Market (1)	Price/ Index# (2)	Log return (3)	Squared log return (4)
Delhi	11201.00***	357649.00***	248169771.00***
Lasalgaon	10887.00***	196230.00***	185100133.00***
Bengaluru	87902.00***	96426.00***	1008184330.00***
S&P 500	347.53***	24828.00***	8035816.00***

****p*<0.01, #for S&P 500

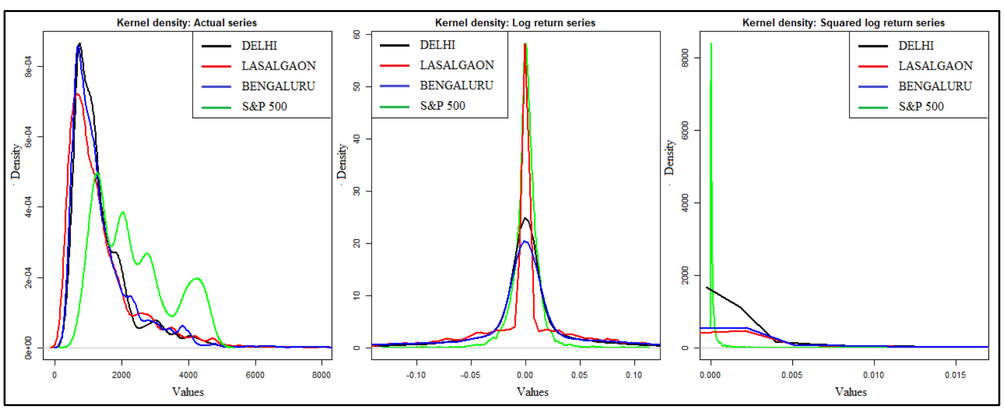

Figure 2: Kernel Density Plots for the Actual Series, Log Return Series, and Squared Log Return Series

3.4 Test for Stationarity

Stationarity of the underlying time series is a prerequisite for the GARCH modeling. The augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979) and the Phillips-Perron (PP) test (Phillips & Perron, 1988) have been used to check the stationarity of the selected series' log return series and their squared log return series. For the ADF and PP tests, the null hypothesis is H_0: The unit root is present in the time series; against the alternative hypothesis H_1: The unit root is not present in the time series. It has been seen that (Table 3) both the tests are significant, and the null hypothesis has been rejected for all tested series. Since all log return series met the stationarity assumption, no additional differentiation has been performed.

TABLE 3: TEST FOR STATIONARITY

Market (1)	Delhi (2)	Lasalgaon (3)	Bengaluru (4)	S&P 500 (5)
Series	Log return	Log return	Log return	Log return
ADF	-17.43	-16.65	-16.76	-16.06
	(0.01)	(0.01)	(0.01)	(0.01)
PP	-85.62	-89.42	-81.89	-72.26
	(0.01)	(0.01)	(0.01)	(0.01)
Series	Squared log return	Squared log return	Squared log return	Squared log return
ADF	-15.53	-16.49	-16.34	-9.07
	(0.01)	(0.01)	(0.01)	(0.01)
PP	-70.44	-57.57	-74.77	-54.95
	(0.01)	(0.01)	(0.01)	(0.01)

p-values are in parenthesis

3.4 Test for Long Memory

To check the long-term persistence among the realizations of the selected log return series and the squared log return series, the GPH test (Geweke & Porter-Hudak, 1983) was performed (Table 4). The estimates of the fractional differencing parameters for the specified log return series are not significant. However, the estimates of the fractional differencing parameters are significant for their squared log return series. This means long-term persistence exists in the squared log return series but not in the log return series. As a result, the prevalence of long-term persistence in volatility has been verified.

TABLE 4: GPH TEST

Market Series (1)	Delhi Log return (2)	Lasalgaon Log return (3)	Bengaluru Log return (4)	S&P 500 Log return (5)
d	-0.019	-0.061	-0.013	-0.017
s.e.	0.056	0.057	0.052	0.075
Z	-0.338	-1.080	-0.252	-0.230
Series	Squared log return	Squared log return	Squared log return	Squared log return
d	0.288	0.079	0.077	0.302
s.e.	0.069	0.020	0.026	0.074
Z	4.173	3.898	2.946	4.084

s.e. denotes standard error.

3.6 Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) Plots

The ACF and PACF plots of the chosen log return series and the ACF plots of the squared log return series are given in Figure 3. The dotted lines in these figures represent the test statistic’s 95 per cent critical values. From the ACF and PACF plots of the log return series, it has been seen that they are decaying at exponential rates. It suggests that the mean model lacks long-term persistence. However, hyperbolic decay is particularly visible in the ACF plots of Delhi's squared log return series and the S&P 500 index. Significant autocorrelations have also been seen at distant lags of the ACF plots of the squared log return series of the Lasalgaon and Bengaluru markets. It suggests the presence of long-term persistence in the volatility. The findings of the GPH test validate the same conclusions.

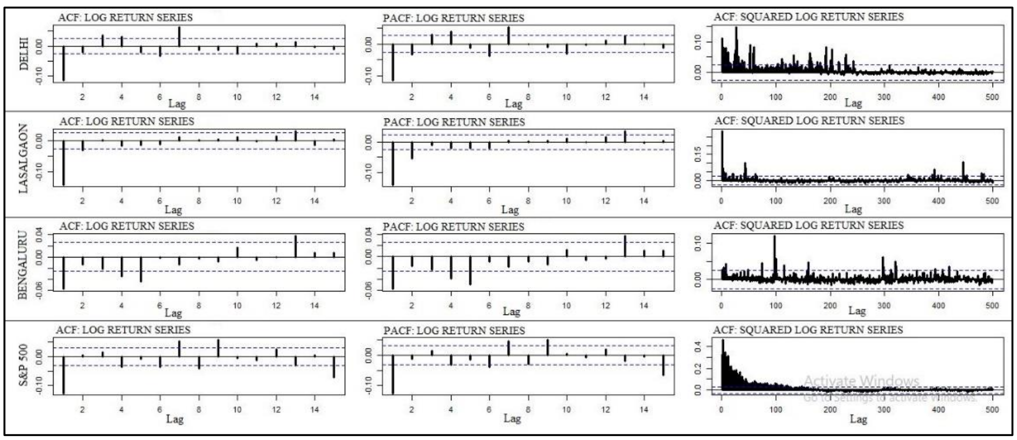

Figure 3: ACF and PACF plots

3.7 Fitting of Models

In the first step, ARMA models with different orders have been fitted as mean models. The residuals have been obtained and tested for conditional heteroscedasticity using the ARCH-LM test. The null hypothesis for this test is H_0: There is no ARCH effect in the residual series; against the alternative hypothesis H_1: There is an ARCH effect in the residual series. It is observed that this test is significant for all residual series, and the null hypothesis has been rejected. After confirming the presence of conditional heteroscedasticity, GARCH, EGARCH, FIGARCH, and FIEGARCH models have been fitted to the residual series. Here, the GARCH and EGARCH models are used as a candidature for symmetric and asymmetric variance models, and FIGARCH and FIEGARCH models are the corresponding models' fractionally integrated versions for capturing long memory in volatility. The best-performed ARMA order has been chosen based on minimum values of the Akaike information criterion (AIC) and Bayesian information criterion (BIC). After selecting the appropriate ARMA order from each of the variance models for all the time series, the best-fitted models have been chosen based on the degree of fitting in terms of three popularly used error functions, namely Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). These error functions have been defined as

$$RMSE = \left[\frac{1}{k}\sum_{t=1}^{k}(y_t - \hat{y}_t)^2\right]^{1/2} \quad ....(13)$$

$$MAE = \frac{1}{k}\sum_{t=1}^{k}|y_t - \hat{y}_t| \quad .... (14)$$

$$MAPE = \frac{1}{k}\sum_{t=1}^{k}\frac{|y_t - \hat{y}_t|}{y_t} \times 100 \quad .... (15)$$

where k denotes the number of realizations used, y_t is the observed value and $\hat{y}_t$ is the corresponding predicted value.

The estimated parameters of the best-fitted models have been given in Table 5. The quasi-maximum likelihood estimation procedure has been carried out for parameter estimation. It has been seen that the FIEGARCH model is the best-fitted model for all the selected time series. From the parameter α_1, it can be inferred that the dependencies of current volatility on the previous shock are insignificant for Delhi and Bengaluru markets. For the remaining two time series, they are significant. The parameter β_1 indicates that the dependencies of current volatility on previous volatility are significant for all of the time series. The asymmetric parameter γ is significant for all of the selected time series. The fractional differencing parameter d is also significant for all instances. From the significant estimates of asymmetric parameters and fractional differencing parameters, it can be inferred that the volatility of selected time series exhibits long-term persistence and is asymmetric. The plots of actual values vs. fitted values in the model building set and forecasted values in the model validation set

TABLE 5: ESTIMATE OF PARAMETERS OF THE BEST-FITTED MODELS

Model Variable	Delhi ARMA (2,0) - FIEGARCH (1, d, 1)	Lasalgaon ARMA (1,0) - FIEGARCH (1, d, 1)	Bengaluru ARMA (1,0) - FIEGARCH (1, d, 1)	S&P 500 ARMA (1,0) - FIEGARCH (1, d, 1)
		Mean Model		
Constant	0.000 (0.000)	-0.000 (0.000)	0.000 (0.000)	0.000 (0.000)***
AR(1)	0.000 (0.000)***	-0.000 (0.000)***	0.001 (0.000)***	-0.060 (0.017)***
AR(2)	0.000 (0.000)***			
		Variance Model		
Constant	-0.474 (0.085)***	0.066 (0.001)***	-0.032 (0.007)***	-0.003 (0.979)
α_1	0.519 (0.525)	0.164 (0.028)***	0.413 (4.829)	-1.002 (0.001)***
β_1	0.736 (0.058)***	0.597 (0.129)***	0.663 (0.105)***	1.000 (0.000)***
θ	0.148 (0.031)***	0.118 (0.003)***	0.122 (0.026)***	0.312 (0.032)***
γ	0.746 (0.144)***	0.437 (0.206)**	0.329 (0.041)***	-0.259 (0.022)***
d	0.657 (0.076)***	0.396 (0.086)***	0.661 (0.067)***	0.792 (0.111)***

***p<0.01, **p<0.05, *p<0.10

have been displayed in Figure 4. Fitting performances in the model building set, in terms of the above-mentioned error functions for the primarily selected variance models, have been given in Table 6. Out of the three criteria, the model that attains minimum values for at least two criteria is considered the best-fitted model.

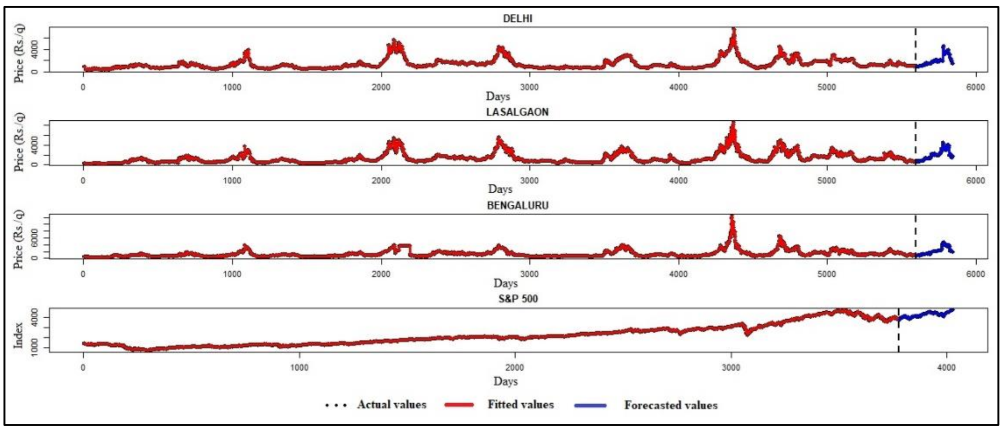

Figure 4: Plots of Actual Values Vs. Fitted Values in the Model Building Set and Forecasted Values in the Model Validation Set of the Finally Selected Models for Onion Data and S&P 500 Index

TABLE 6: FITTING PERFORMANCE OF THE SELECTED MODELS IN THE MODEL BUILDING SET

Market	Model	RMSE	MAE	MAPE (%)
Delhi	ARMA (0,2) -GARCH (1,1)	102.199	36.946	2.633
	ARMA (2,0) -EGARCH (1,1)	**102.004**	36.943	2.629
	ARMA (0,1) -FIGARCH (1, d, 1)	102.152	36.945	2.631
	ARMA (2,0) -FIEGARCH (1, d, 1)	102.013	**36.941**	**2.624**
Lasalgaon	ARMA (0,1) -GARCH (1,1)	**158.309**	62.433	4.067
	ARMA (1,0) -EGARCH (1,1)	159.938	59.657	3.872
	ARMA (0,1) -FIGARCH (1, d, 1)	159.937	59.652	3.867
	ARMA (1,0) -FIEGARCH (1, d, 1)	159.939	**59.651**	**3.861**
Bengaluru	ARMA (1,1) -GARCH (1,1)	152.307	55.993	3.838
	ARMA (1,1) -EGARCH (1,1)	**151.481**	52.643	3.631
	ARMA (0,2) -FIGARCH (1, d, 1)	152.944	54.921	3.766
	ARMA (1,0) -FIEGARCH (1, d, 1)	151.482	**52.642**	**3.630**
S&P 500	ARMA (1,0) -GARCH (1,1)	29.028	17.618	0.834
	ARMA (1,0) -EGARCH (1,1)	29.007	**17.613**	0.834
	ARMA (0,1) -FIGARCH (1, d, 1)	29.032	17.619	0.834
	ARMA (1,0) -FIEGARCH (1, d, 1)	**28.964**	17.624	**0.833**

The forecasting performance for the model validation set has been examined using a rolling window forecast over 50 days, 100 days, 150 days, 200 days, and 250 days. The forecasting performance of the selected FIEGARCH models for each time series on the aforementioned rolling window basis is presented in Table 7. It has been observed that for all the onion price series and the S&P 500 index, forecasting performance improves with an increase in the forecast horizons of different extend. This improvement is attributed to the long-term persistence of volatility. Long memory in volatility helps improve forecasting performance for the longer horizon.

TABLE 7: ROLLING WINDOW FORECASTING PERFORMANCE OF FINALLY SELECTED MODELS IN THE MODEL VALIDATION SET

Market	Horizon	RMSE	MAE	MAPE (per cent)
(1)	(2)	(3)	(4)	(5)
Delhi	50	22.034	5.600	0.533
	100	47.813	14.420	1.113
	150	159.800	36.321	1.930
	200	114.518	29.462	1.759
	250	104.147	16.562	1.254
Lasalgaon	50	76.479	51.780	6.285
	100	68.312	46.760	4.862
	150	66.235	43.646	4.129
	200	155.529	72.157	7.134
	250	191.469	85.484	7.420
Bengaluru	50	75.828	31.000	3.008
	100	69.731	29.500	2.480
	150	68.007	28.667	2.112
	200	115.786	41.250	3.902
	250	129.865	49.400	4.258
S&P 500	50	41.027	33.191	0.831
	100	37.386	29.222	0.725
	150	33.985	26.185	0.635
	200	33.694	26.171	0.625
	250	33.301	25.622	0.606

Plotting the ACF and PACF of the residuals allows us to assess the suitability of the best-fitted FIEGARCH models for all the chosen time series. The residuals exhibited no discernible systematic trend that might be further explained. Almost all the correlations fall within the 95 per cent confidence interval. Hence, it can be concluded that the best-fitted FIEGARCH models effectively captured the long-term persistence of asymmetric volatility present in the selected time series.

IV

CONCLUSIONS

A better understanding of the fluctuations in agricultural commodity prices can significantly empower the farming community. Onion prices, in particular, are highly sensitive to controllable and uncontrollable factors. This research paper investigates the presence of long memory and asymmetric volatility in onion price series. Among the GARCH, EGARCH, FIGARCH, and FIEGARCH models, the FIEGARCH model emerges as the best-fitted model for the daily modal price series of onions in Delhi, Lasalgaon, and Bengaluru markets, as well as for the S&P 500 index (close) data. The findings confirm the existence of asymmetric volatility and long-term persistence in volatility for these selected time series. Long-term persistence in volatility enhances the model's forecasting efficiency over different extended horizons. Addressing the long memory property of volatility in any time series can provide valuable insights for understanding its behaviour, facilitating informed decision-making, and optimizing resource utilisation. In summary, this study highlights the presence of long memory and asymmetric volatility in analysing onion price dynamics. The insights gained from this research can aid farmers, policymakers, and market participants in making more informed decisions, ultimately benefiting the agricultural sector and the broader economy.

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